Modelling Legal Relations

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Abstract

We use propositional dynamic logic and ideas about propositional control from the agency literature to construct a simple model of how legal relations interact with actions that change the world, and with actions that change the legal relations.

Our conceptual model also allows us to study the interplay of obligation, knowledge, and ignorance, and to model knowledge based obligation.

This work is relevant for attempts to construct restricted fragments of natural language for legal reasoning that could be used in the creation of (more) formal versions of legal documents suitable for 'legal knowledge bases'.

(this is joint work with Fengkui Ju, Beijing Normal University, Beijing, China)

Overview

- What Ethics is About; What the Law is About
- Legal Claims, Duties, Etc. Action and Forbearance
- First Simplify, then Model
- Legal Relations and Interest in Taking Legal Action
- Knowledge and Ignorance
- Dynalex Language, Models, Truth, Action Interpretation
- Examples (Knowledge and Ignorance of Action Utilities, Knowledge Based Obligation, Obligation to Repair Damage, Contrary to Duty Obligation, Contract Exchange)
- Permissions, Obligations, Legal Acceptability

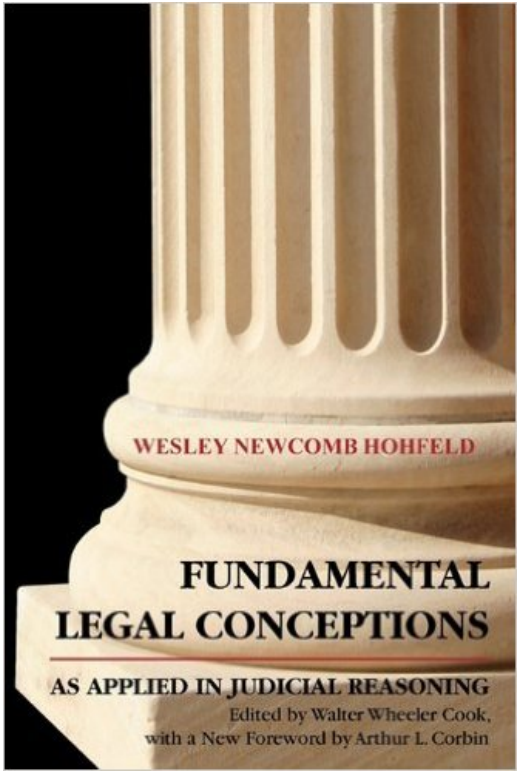
- Towards a Calculus for Dynalex, Agenda for Future Work

Morality is About Action

The Law is About Relations between People Regarding Actions

Wesley Newcomb Hohfeld, 1879–1918

American jurist, famous for Fundamental Legal Conceptions
[Hoh13, Hoh20]



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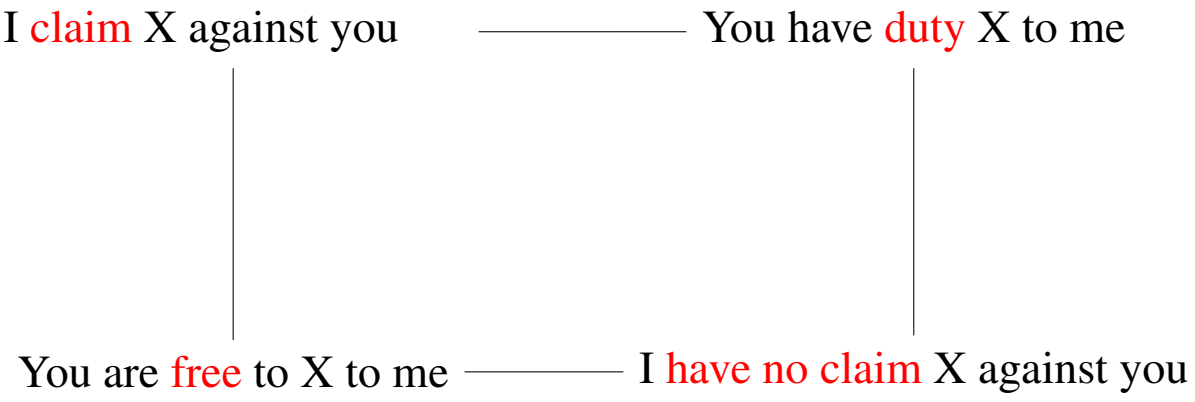
Jural Opposites (Related by Negation)

- Right versus No-right (or: Claim versus No-claim)
- Privilege versus Duty (or: Liberty versus Duty)
- Power versus Disability
- Immunity versus Liability

Jural Correlatives (Related by Swap of Agent Role)

- Right versus Duty
- Privilege versus No-right
- Power versus Liability
- Immunity versus Disability

Square of Opposition for Legal Rights/Claims



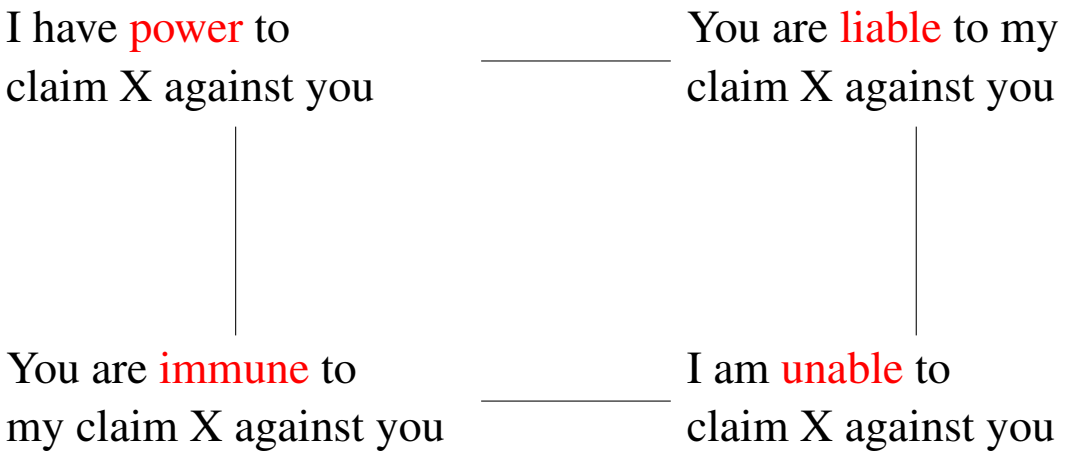
What is X?

Hohfeld: X = "to act or forbear for the benefit of another person"

Example Case of Jan van Eijck (JvE) Versus Shalom Lappin (SL)

- JvE spots a note of 1000 kronor on the pavement in Gothenburg.
- JvE sees SL approaching; SL also spots the banknote.
- JvE and SL are both at liberty to grab it.
- JvE quickly picks up the note and puts it in his pocket.
- JvE has established a claim of ownership.
- Others (including SL) have incurred a duty to respect this.
- SL has no claim against JvE.

Square of Opposition for Legal Powers



Example Case of Jan van Eijck Versus a Stranger

- JvE puts some piece of old furniture out with his garbage.
- By that act JvE is giving up a claim of ownership.
- JvE has the power to do this, for he was the rightful owner.
- A stranger finds the old chair on the street and says to himself, 'hey, I can still use this'.
- The finder is establishing a claim of ownership.
- Putting these acts together we see that an old chair now has a new owner.

Hohfeld in His Own Words (or those of A.L. Corbin [Cor19])

Right, Claim "An enforceable claim to performance (action or forbearance) by another. It is the legal relation of A to B when society commands action or forebearance by B and will at the instance of A in some manner penalize disobedience."

Duty "It is the legal relation of a person, B, who is commanded by society to act or to forbear for the benefit of another person, A, either immediately or in the future, and who will be penalized by society for disobedience."

Privilege, Liberty "The legal relation of A to B when A (with respect to B) is free or at liberty to conduct himself in a certain manner for the benefit of B by the command of society; and when he is not threatened with any penalty for disobedience, for the reason that society has made no command."

No-right, No-claim "The legal relation of a person (A) in whose behalf society commands nothing of another (B)."

Power "The legal relation of A to B when A's own voluntary act will cause new legal relations either between B and A or between B and a third person."

Liability "The relation of A to B when A may be brought into new legal relations by the voluntary act of B."

Immunity "The relation of A to B when B has no legal power [...] to affect some one or more of the existing legal relations of A. As to that particular existing relation A has an immunity with respect to B."

Disability "The relation of A to B when by no voluntary act of his own can A extinguish one (or more) of the existing legal relations of B."

Action

"To act or forbear"

forbear = politely or patiently restrain an impulse to do something; refrain.

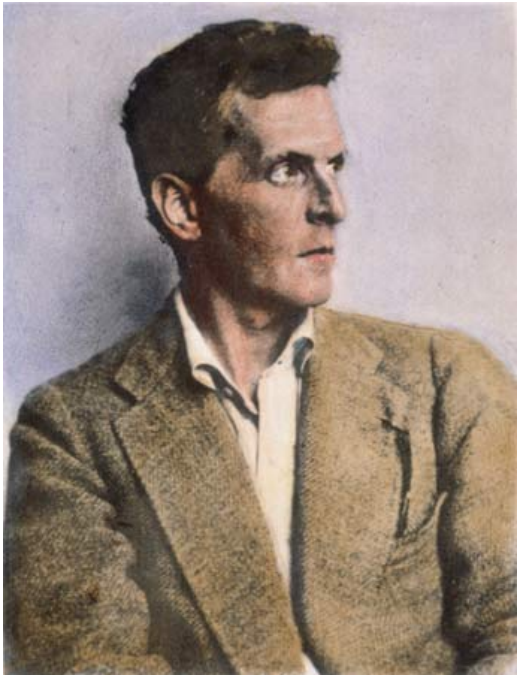
To act: to change certain basic facts of the world by one's power of agency.

To forbear: to restrain an impulse to prevent an agent from changing certain basic facts of the world within the power of that agent.

Programme

- interpret to act and to forbear in the simplest possible way,
- incorporate these interpretations in a formal system,
- investigate the expressive power and properties of the system.

Philosophical Simplification



"Die Welt ist die Gesamtheit der Tatsachen, nicht der Dinge."

What is a Basic Action?

Change a basic proposition from true to false or vice versa:

$$p := \neg p$$

Actions are performed by agents. We assume that each agent has agentive power over a subset of the set of all basic proposition letters.

This gives:

$$(a, p := \neg p)$$

It is assumed that a has agentive power over p.

What is To Forbear?

For a to forbear acting on p is not to perform action

$$(a, p := \neg p)$$

It is assumed that a has agentive power over p.

Claims Tables: Claimed Actions

Let P be a set of basic propositions.

$C^{+}_{ab}(w) \subseteq P$ are the actions that are claimed in w by a against b, in the sense that a claims that b has the duty to perform them.

Example: a is owner, b is tenant, and $p := \neg p$ is the action of paying the rent.

Claims Tables: Claimed ForBearances

$C^-_{ab} \subseteq P$ are the propositions that are claimed in w as forbearances by a against b, in the sense that a claims that b has the duty to forbear operating on p.

If $p \in C^-_{ab}(w)$, then by forbearance is meant that b should not flip the truth value of p.

Example: a claims that b as a tenant should forbear property inspections by a. That is, a is owner, b is tenant, and $p := \neg p$ is the action of property inspection.

It is claimed by a that b should not interfere when a carries out such actions.

Consistency Requirements on Claims Tables

- $C^{+}_{ab}(w) \cap C^{-}_{ab}(w) = \emptyset$

  if in w b has the duty towards a of operating on p then in w b cannot have the duty towards a of forbearing on p

- $C^{+}_{ab}(w) \cap C^{+}_{ba}(w) = \emptyset$

  if in w b has a duty towards a to an action, then a does not have a duty towards b to perform the same action

- $C^{-}_{ab}(w) \cap C^{-}_{ba}(w) = \emptyset$

  if in w, b has a duty towards a to forbear, then in w a does not have the same duty towards b to forbear

Towards a Logical Language

In the logical language we need propositions for basic claims and duties. We will use:

- $C^+(a, b, p)$ for "a claims the right against b that b should operate on p",
- $D^+(b, a, p)$ for "b has the duty towards a to operate on p",
- $C^-(a, b, p)$ for "a claims the right against b that b should not act on p",
- $D^-(b, a, p)$ for "b has the duty towards a that b should not act on p".

Legal Powers

If we want to express legal powers, we think that (in our simplified version of the legal framework) the following is meant: the power to change the C^{+}_{ab} and C^{-}_{ab} relations.

Example: the furniture example above.

Conceptual Analysis: Claims

- a has a claim against b about b setting the value of p, or: a claims against b that b has the duty of setting the value of p.
- $C^+(a, b, p)$ with semantics $p \in C^+_{ab}(w)$
- a has a claim against b about b forbearing actions on p, or: a claims against b that b has the duty to refrain from setting the value of p.
- $C^-(a, b, p)$ with semantics $p \in C^-_{ab}(w)$

Conceptual Analysis: Claims Versus Duties

- b has the duty towards a of refraining from settting the value of p (or: not to interfere when a operates on p).
- $D^-(b, a, p)$ with semantics $p \in C^-_{ab}(w)$.
- b has the duty towards a to act on p.
- $D^+(b, a, p)$ with semantics $p \in C^+_{ab}(w)$.

Claims and Duties are Correlates

- $C^+(a, b, p) \leftrightarrow D^+(b, a, p)$.
- $C^-(a, b, p) \leftrightarrow D^-(b, a, p)$.

Conceptual Analysis: Privileges or Liberties

- a has the liberty against b of operating on p. This has the same meaning as: a does not have a duty towards b of refraining from operating on p.
- $\neg D^{-}(a, b, p)$ with semantics $p \notin C^{-}_{ba}(w)$.
- a has the liberty against b of not operating on p. This has the same meaning as: a does not have a duty towards b of operating on p.
- $\neg D^{+}(a, b, p)$ with semantics $p \notin C^{+}_{ba}(w)$.

Conceptual Analysis: No-Claims

- b has no claim against a about operating on p.
- $\neg C^{+}(b, a, p)$ with semantics $p \notin C^{+}_{ba}(w)$.
- b has no claim against a about refraining from p.
- $\neg C^{-}(b, a, p)$ with semantics $p \notin C^{-}_{ba}(w)$.

Liberties and No-Claims are Correlates

- $\neg D^{+}(a, b, p) \leftrightarrow \neg C^{+}(b, a, p)$.
- $\neg D^{-}(a, b, p) \leftrightarrow \neg C^{-}(b, a, p)$.

Basic Actions for Executing Legal Powers

- $C^+(a, b, -p)$, with semantics $C^+_{ab}(w) := C^+_{ab}(w) - \{p\}$.
  This expresses that a gives up a claim against b that b should act on p.
- $C^-(a, b, -p)$, with semantics $C^-_{ab}(w) := C^-_{ab}(w) - \{p\}$.
  This expresses that a gives up a claim against b that b should forbear actions on p.
- $C^+(a, b, +p)$, with semantics $C^+_{ab}(w) := C^+_{ab}(w) \cup \{p\}$.
  This expresses that a establishes a claim against b that b should act on p.
- $C^-(a, b, +p)$, with semantics $C^-_{ab}(w) := C^-_{ab}(w) \cup \{p\}$.
  This expresses that a establishes a claim against b that b should forbear actions on p.

Actions for Executing Legal Powers, As Duty Changes

- $D^{+}(a, b, -p)$ for $C^{+}(b, a, -p)$,
- $D^{-}(a, b, -p)$ for $C^{-}(b, a, -p)$,
- $D^{+}(a, b, +p)$ for $C^{+}(b, a, +p)$,
- $D^{-}(a, b, +p)$ for $C^{-}(b, a, +p)$.

Correlates in the Area of Legal Powers

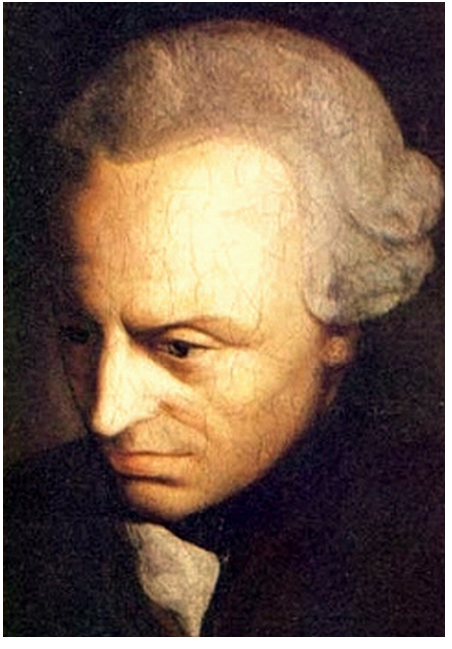
Powers and Liabilities are Correlates

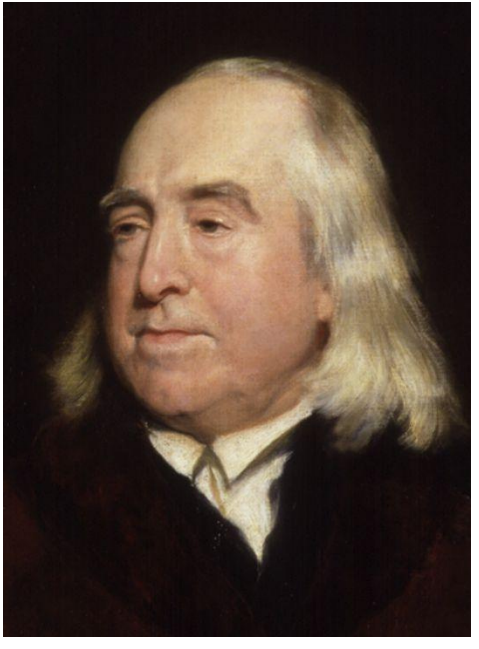
- $\langle C^{+}(a, b, +p)\rangle\varphi \leftrightarrow \langle D^{+}(b, a, +p)\rangle\varphi$
- $\langle C^{-}(a, b, +p)\rangle\varphi \leftrightarrow \langle D^{-}(b, a, +p)\rangle\varphi$
- $\langle C^{+}(a, b, -p)\rangle\varphi \leftrightarrow \langle D^{+}(b, a, -p)\rangle\varphi$
- $\langle C^{-}(a, b, -p)\rangle\varphi \leftrightarrow \langle D^{-}(b, a, -p)\rangle\varphi$

Immunities and Disabilities are Correlates

- $\neg\langle D^{+}(a, b, +p)\rangle\varphi \leftrightarrow \neg\langle C^{+}(b, a, +p)\rangle\varphi$
- $\neg\langle D^{-}(a, b, +p)\rangle\varphi \leftrightarrow \neg\langle C^{-}(b, a, +p)\rangle\varphi$
- $\neg\langle D^{+}(a, b, -p)\rangle\varphi \leftrightarrow \neg\langle C^{+}(b, a, -p)\rangle\varphi$
- $\neg\langle D^{-}(a, b, -p)\rangle\varphi \leftrightarrow \neg\langle C^{-}(b, a, -p)\rangle\varphi$

Bringing in a Utilitarian Element





Immanuel Kant Versus Jeremy Bentham

Adding Utilities

A payoff function or utility function U is a map from agents to functions from valuations to values in the real numbers.

$$U : I \to \mathcal{P}(P) \to \mathbb{R}$$

Use U_a for the payoff function for agent a.

The payoff for agent a of an action of changing the value of p can be computed as the difference

$$U_a(V') - U_a(V)$$

where V is the old valuation and V' the new valuation (after the update of the value of p).

Computing the Societal Payoff of an Action

The societal payoff of an action that changes V to V' can be computed as

$$\sum_{a \in I} U_a(V') - \sum_{a \in I} U_a(V)$$

Interest in Taking Legal Action

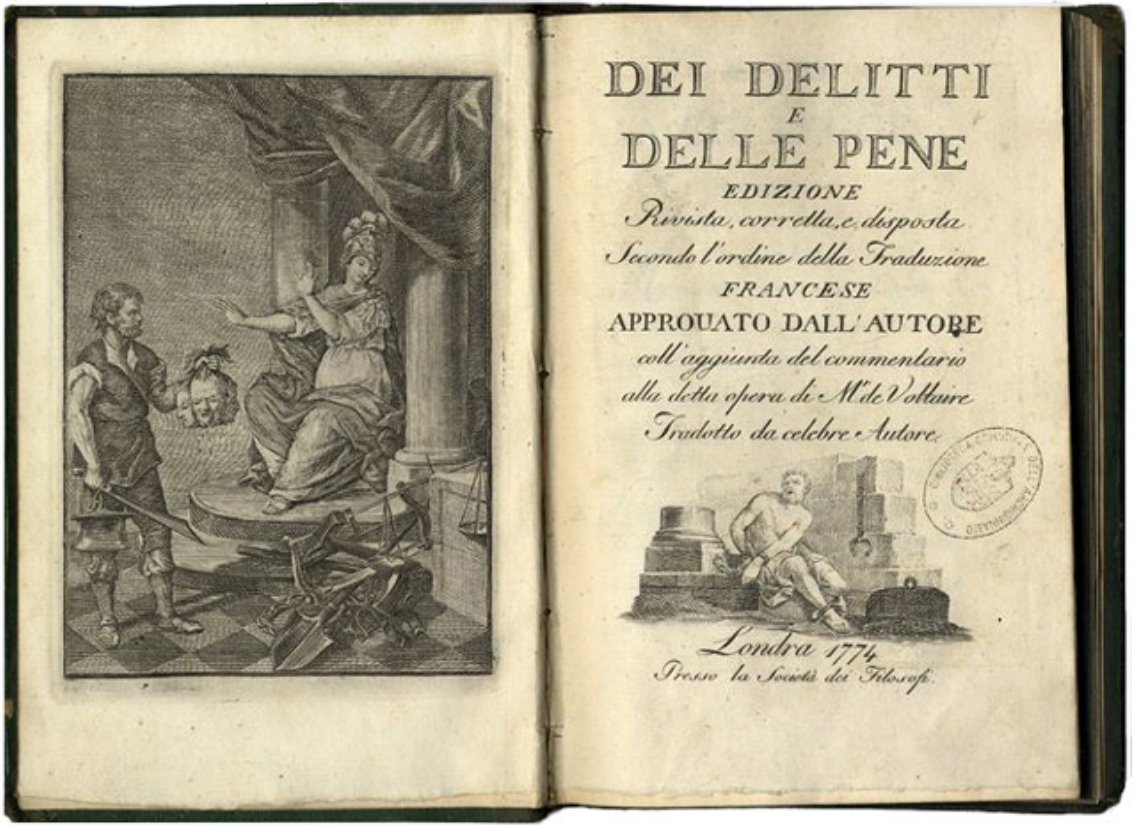
Actions may have benefits for one agent and costs for another agent, benefits for society or costs for society.

How do these affect the legal relations? If we have a payoff function, then we can formalize the notion of agent a having an interest to take legal action against agent b.

The fact that someone infringes on my rights is one element in this.

The other element is the fact that someone causes harm to me by his action, in the very precise sense of decreasing my individual payoff.

Cesare Beccaria (1738 – 1794)



DEI DELITTI
E
DELLE PENE
EDIZIONE
Rivista, corretta, e disposta
Secondo l'ordine della Traduzione
FRANCESE
APPROVATO DALL'AUTORE
coll'aggiunta del commentario
alla detta opera di M.r de Voltaire
Tradotto da celebre Autore.

Londra 1774.
Presso la Società dei Filosofi.

On Crimes and Punishment [Bec64]

Beccharia on Proportionality of Punishment

> If an equal punishment be ordained for two crimes that injure society in different degrees, there is nothing to deter men from committing the greater as often as it is attended with greater advantage. [Bec64, Ch 6]

Here is a second level where utilities come in. Society imposes a punishment on those who transgress the law, and this punishment can be viewed as a negative utility.

For now, let us keep this second level of utilities out of the picture: utilities are the utilities of the actions per se, irrespective of possible societal sanctions.

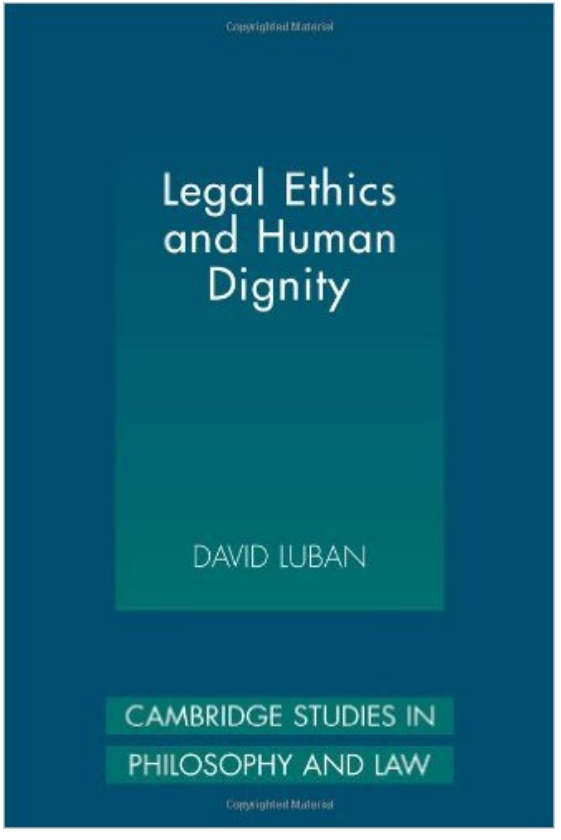
Many transgressions of the law go unpunished, and for a sophisticated treatment one would also have to take the probability of getting caught in a crime into account [Eij15].

Knowledge and Ignorance

Issues of knowledge and ignorance of the facts are crucial in moral and legal reasoning; see, e.g., the treatment of knowledge based obligation in [PPC06].

Compare also the following revealing quote from [Lub07, Ch 6]:

> The fact is that ignorance can be vital. A white-collar defense attorney offers the following recollection: "I can remember years ago when I represented a fellow in a massive case of political corruption. I was very young, and I asked him, 'Would you please tell me everything that happened.' And he said, 'What, are you out of your mind?'"



[Lub07, Ch 6] on Contrived Ignorance

The DynaLex Language $\mathcal{L}$

Let a range over a set of agents I and p over a set of propositions P.

$$\begin{aligned}
\varphi &::= \top \mid p \mid A(a,p) \mid C^+(a,b,p) \mid C^-(a,b,p) \mid \\
&\quad\ \sigma \leq_a \sigma \mid \neg\varphi \mid \varphi \vee \varphi \mid \Diamond_a\varphi \mid \langle\alpha\rangle\varphi. \\
A &::= (a, p := \neg p) \mid C^+(a,b,+p) \mid C^+(a,b,-p) \mid \\
&\quad\ C^-(a,b,+p) \mid C^-(a,b,-p) \\
\sigma &::= [A,\ldots,A] \\
\alpha &::= \sigma \mid ?\varphi \mid \alpha;\alpha \mid \alpha \cup \alpha \mid \alpha^*
\end{aligned}$$

Condition on σ: all actions have unique agents.

Usual definitions of $\bot$, $\wedge$, $\rightarrow$, $\leftrightarrow$ and of the duals $\Box_a$ of $\Diamond_a$ and $[\alpha]$ of $\langle\alpha\rangle$.

Def of duties in terms of claims: $D^{\pm}(a,b,p)$ for $C^{\pm}(b,a,p)$, etc.

DynaLex Models

A DynaLex model M for a set of propositions P and a set of agents I is a tuple $M = (W, R, V, A, C, U)$ where

- W is a non-empty set of worlds,
- $R : I \to \mathcal{P}(W^2)$ is a function that maps each agent a to an equivalence relation $R(a) = \sim_a$ on W,
- $V : W \to \mathcal{P}(P)$ is a valuation function,
- $A : I \to \mathcal{P}(P)$ is an ability table, listing for each agent the propositions that the agent is able to operate upon,
- $C : I^2 \to \mathcal{P}(P) \to (\mathcal{P}(P), \mathcal{P}(P))$ is a claims table,
- $U : I \to \mathcal{P}(P) \to (I^2 \to \mathcal{P}(P) \to (\mathcal{P}(P), \mathcal{P}(P))) \to \mathbb{R}$ is a utility function.

Substitution for $\mathcal{L}$ Atoms

$$
\begin{aligned}
\sigma(p) &= \begin{cases} (A(a,p) \wedge \neg p) \vee (\neg A(a,p) \wedge p) & \text{if } (a, p := \neg p) \in \sigma \\ p & \text{otherwise} \end{cases} \\
\sigma(A(a,p)) &= A(a,p) \\
\sigma(C^+(a,b,p)) &= \begin{cases} \bot & \text{if } C^+(a,b,-p) \in \sigma \\ \top & \text{if } C^+(a,b,+p) \in \sigma \\ C^+(a,b,p) & \text{otherwise} \end{cases} \\
\sigma(C^-(a,b,p)) &= \begin{cases} \bot & \text{if } C^-(a,b,-p) \in \sigma \\ \top & \text{if } C^-(a,b,+p) \in \sigma \\ C^-(a,b,p) & \text{otherwise} \end{cases}
\end{aligned}
$$

Model Substitution

Let $M = (W, R, V, A, C, U)$ be a DynaLex model and let σ be a $\mathcal{L}$ substitution. Then V^σ is the valuation given by:

$$V^\sigma(w) = \{p \in P \mid M, w \models \sigma(p)\}.$$

C^σ is the claims table of which the left side is given by:

$$\text{left}(C^\sigma(a,b)(w)) = \{p \in P \mid M, w \models \sigma(C^+(a,b,p))\},$$

and the right side by:

$$\text{right}(C^\sigma(a,b)(w)) = \{p \in P \mid M, w \models \sigma(C^-(a,b,p))\}.$$

M^σ is given by $(W, R, V^\sigma, A, C^\sigma, U)$.

Truth

Let $M = (W, V, A, C, L, U)$ be a DynaLex model, let $w \in W$. Then:

$$
\begin{array}{rll}
M, w \models \top & & \text{always} \\
M, w \models p & \text{iff} & p \in V(w) \\
M, w \models A(a, p) & \text{iff} & p \in A_a \\
M, w \models C^+(a, b, p) & \text{iff} & p \in C^+_{ab}(w) \\
M, w \models C^-(a, b, p) & \text{iff} & p \in C^-_{ab}(w) \\
M, w \models \sigma_1 \leq_a \sigma_2 & \text{iff} & U(a)(V_w^{\sigma_1})(C^{\sigma_1}) \leq U(a)(V_w^{\sigma_2})(C^{\sigma_2}) \\
M, w \models \neg\varphi & \text{iff} & \text{not } M, w \models \varphi \\
M, w \models \varphi_1 \vee \varphi_2 & \text{iff} & M, w \models \varphi_1 \text{ or } M, w \models \varphi_2 \\
M, w \models \Diamond_a \varphi & \text{iff} & \text{for some } v \text{ with } (w, v) \in \sim_a\text{: } M, v \models \varphi \\
M, w \models \langle\alpha\rangle\varphi & \text{iff} & \text{for some } (M', w') : (M, w)[\![\alpha]\!](M', w') \\
 & & \text{where } [\![\alpha]\!] \text{ is as given on next slide.}
\end{array}
$$

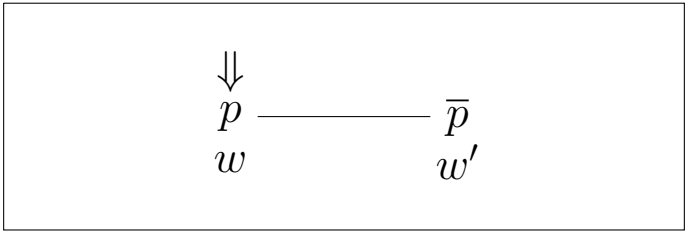
Action Interpretation

This uses the earlier definition of M^σ.

$$
\begin{array}{rll}
(M,w)[\![\sigma]\!](M',w') & \text{iff} & M' = M^\sigma \text{ and } w = w' \\
(M,w)[\![\,?\varphi]\!](M',w') & \text{iff} & (M,w) = (M',w') \text{ and } M,w \models \varphi \\
(M,w)[\![\,\alpha_1;\alpha_2]\!](M',w') & \text{iff} & \text{for some } N, v: \\
 & & (M,w)[\![\,\alpha_1]\!](N,v) \text{ and } (N,v)[\![\,\alpha_2]\!](M',w') \\
(M,w)[\![\,\alpha_1 \cup \alpha_2]\!](M',w') & \text{iff} & (M,w)[\![\,\alpha_1]\!](M',w') \text{ or } (M,w)[\![\,\alpha_2]\!](M',w') \\
(M,w)[\![\,\alpha^*]\!](M',w') & \text{iff} & \text{there are } (N_0,w_0),\ldots,(N_n,w_n) \\
 & & \text{with } (M,w) = (N_0,w_0), \\
 & & (M',w') = (N_n,w_n), \\
 & & \text{and } (N_i,w_i)[\![\,\alpha]\!](N_{i+1},w_{i+1}) \\
 & & \text{for all } i \text{ with } 0 \leq i < n.
\end{array}
$$

Single Agent Example

Assume the vocabulary is $\{p\}$, and let there be a single agent a. We use a solid line for a-accessibility. The $\Downarrow$ arrow points at the actual world.

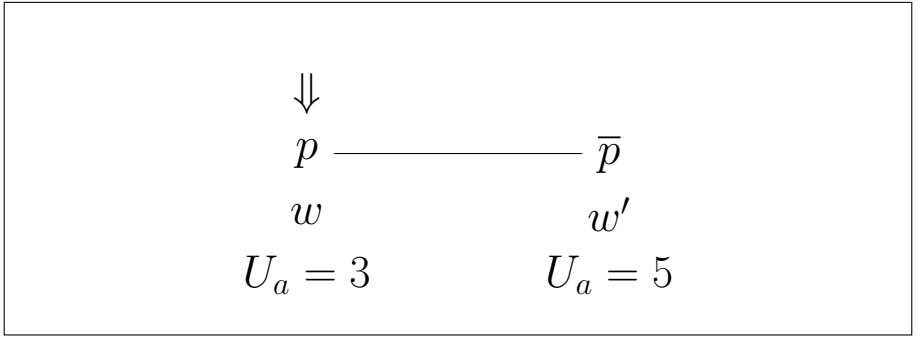


$$A_a = \{p\}$$

$$\begin{aligned} w &\models \neg\Box_a p \\ w &\models [(a, p := \neg p)]\neg\Box_a p \\ w &\models \Box_a[(a, p := \neg p)]\neg\Box_a p \end{aligned}$$

Adding a Utility Function

Assume a utility function that only depends on the valuation argument.

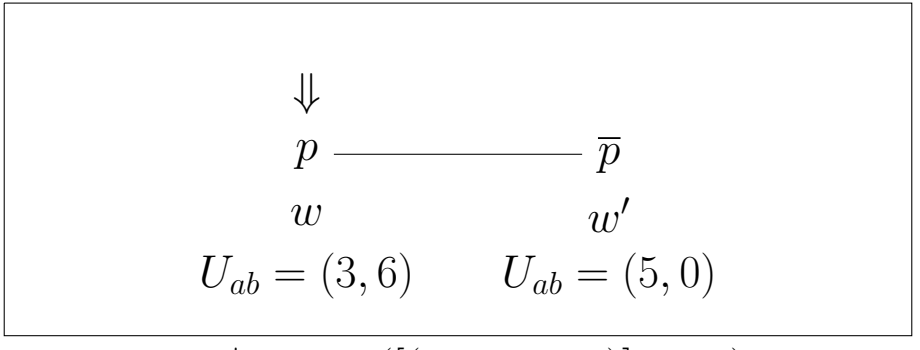


$$
\begin{aligned}
w &\models (a, p := \neg p) >_a \epsilon \\
w' &\models (a, p := \neg p) <_a \epsilon \\
w &\models \neg \Box_a ((a, p := \neg p) >_a \epsilon)
\end{aligned}
$$

In w, a is able to flip the value of p and it is in a's interest to flip the value of p, but a does not know this.

Bringing In Another Agent

Now suppose we also have another agent, b, with $A_b = \{p\}$, that is, b can also operate on p. Assume b knows whether p is true.

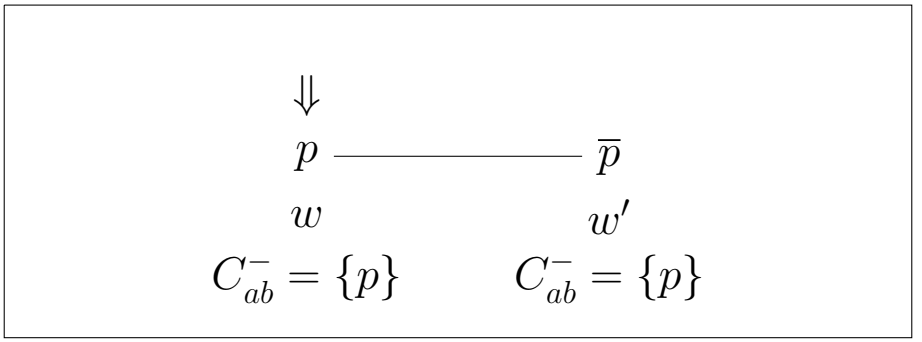


$$w \models \neg\Box_a([(b, p := \neg p)] >_a \epsilon)$$
$$w \models \neg\Box_a([(b, p := \neg p)] <_b \epsilon)$$
$$w \models \Box_b([(b, p := \neg p)] >_a \epsilon)$$
$$w \models \Box_b([(b, p := \neg p)] <_b \epsilon)$$

Global Claims

Suppose agent a has a claim against b that b should forbear on p. That is, suppose that the claims table has $C^-_{ab}(w) = C^-_{ab}(w') = \{p\}$. To be consistent, the claims table should also have $C^-_{ba}(w) = C^-_{ba}(w') = \emptyset$.

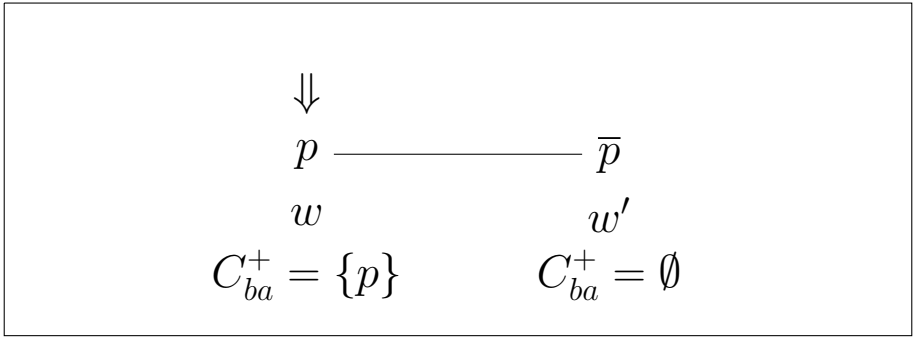
Then $C^-(a, b, p)$ is true everywhere, and the action $(b, p := \neg p)$ is a violation of the claim that a holds against b.



$$A_a = A_b = \{p\}$$

Specific Claims

A claim is specific if it varies with the situation. Assume a solid line for a-accessibility. Assume b can distinguish w and w'. Consider:



$$A_a = \{p\}, A_b = \emptyset$$

In world w, a has a duty towards b to operate on p. In world w', a has no such duty. But a cannot distinguish between the two worlds.

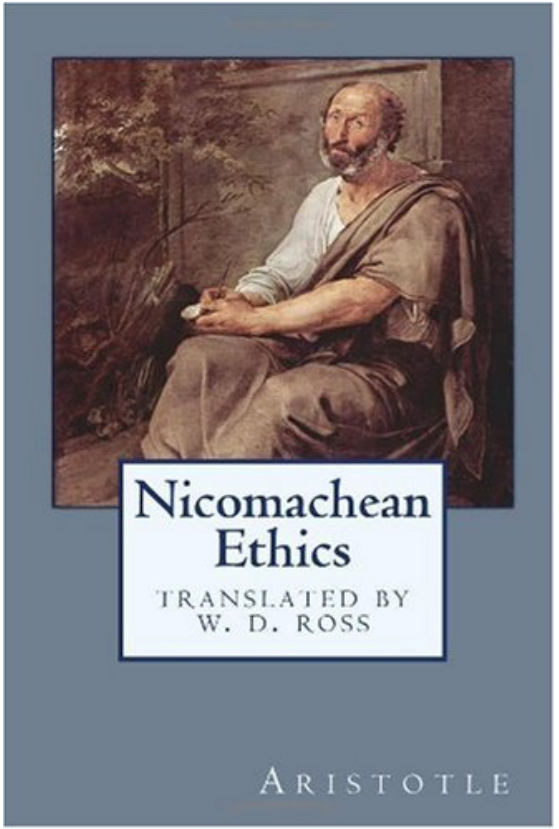
Knowledge Based Obligation

To flesh out the picture, assume a is a doctor, and p means that agent b is in danger. a is able to save b (putting him out of danger), and b cannot save himself.

But a does not know b is in danger, and as a consequence a does not know she is under an obligation to take action.

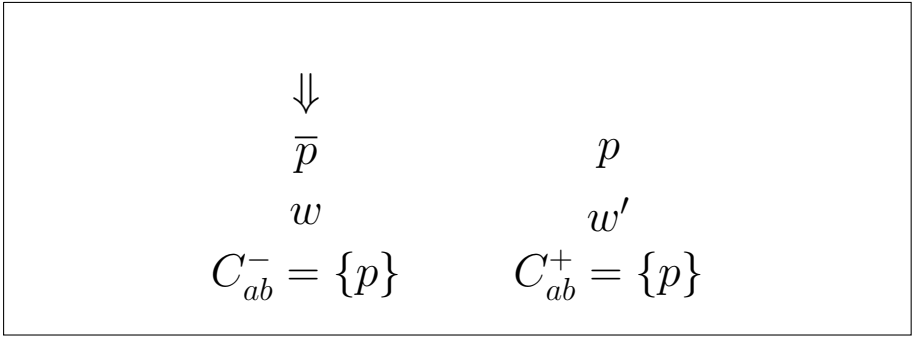
What emerges is a picture of knowledge based obligation. The following are true in the actual world:

$$
\begin{aligned}
w &\models p \\
w &\models \neg \Box_a p \\
w &\models D^+(a, b, p) \\
w &\models \neg \Box_a D^+(a, b, p)
\end{aligned}
$$



Aristotle on Responsibility [Ari80, Book III]

Obligation to Repair Damage

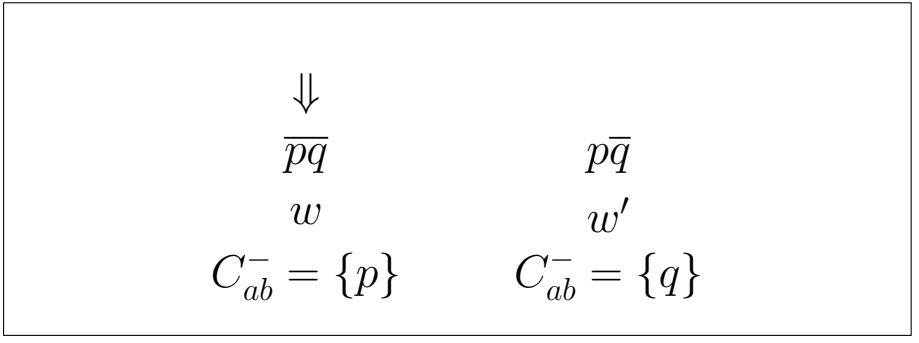


$A_a = \emptyset, A_b = \{p\}$

In the actual world b has the duty to forbear on p. But if b acts contrary to duty by operating on p, then b incurs the new duty of restoring the old situation.

$$w \models C^-(a, b, p)$$
$$w \models [(b, p := \neg p)]C^+(a, b, p)$$

Contrary to Duty Obligation

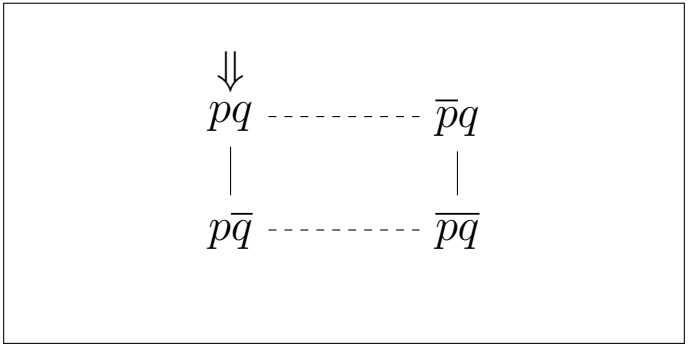


$A_a = \emptyset, A_b = \{p, q\}$

$$w \models C^-(a, b, p)$$
$$w \models [(b, p := \neg p)]C^-(a, b, q)$$

Agent b has the duty towards a of not operating on p, and after b acts contrary to duty by operating on p, b still has the duty towards a of not operating on q.

Agents with Different Knowledge and Different Interests



Assume $A_a = A_b = \{p, q\}$.

pq	$p\overline{q}$	$\overline{p}q$	$\overline{pq}$
(0,0)	(10,0)	(0,10)	(5,5)

Agents a and b have a common interest: avoid a situation where they end up in pq. It makes sense for a to grant b the right to operate on q and for b to grant a the right to operate on p.

Exchange a Contract

pq	$p\overline{q}$	$\overline{p}q$	$\overline{pq}$
(0,0)	(10,0)	(0,10)	(5,5)

$$[C^-(a, b, +p), C^-(b, a, +q)].$$

This gives $C^-_{ab} = \{p\}$ and $C^-_{ba} = \{q\}$. These claims tables are global. Now a can execute $p := \neg p$ and b can execute $q := \neg q$, with a utility of 5 for both as a result.

Defining Permissions and Obligations

Some examples:

$$P_a(a, p := \neg p) \leftrightarrow \neg \bigvee_{b \in I} C^-(b, a, p).$$

$$P_a(\alpha; \beta) \leftrightarrow P_a(\alpha) \wedge [\alpha] P_a \beta.$$

$$P_a(?\varphi; \alpha) \leftrightarrow \varphi \wedge P_a(\alpha).$$

$$P_a(\alpha \cup \beta) \leftrightarrow P_a(\alpha) \wedge P_a(\beta) \wedge \neg O_a(\alpha) \wedge \neg O_a(\beta).$$

$$O_a(a, p := \neg p) \leftrightarrow \bigvee_{b \in I} C^+(b, a, p).$$

Defining Legal Acceptability

When is an action legally acceptable for an agent a? Assume that a is not taking part in the action; this is a different concept from permissibility.

Some examples:

$$A_a(b, p := \neg p) \leftrightarrow \neg C^-(a, b, p).$$

$$A_a(\alpha; \beta) \leftrightarrow A_a(\alpha) \wedge [\alpha] A_a(\beta).$$

$$A_a(\alpha \cup \beta) \leftrightarrow A_a(\alpha) \wedge A_a(\beta).$$

Towards a Calculus for DynaLex

- Axioms expressing the relation of capabilities and duties. An agent cannot have a duty to operate on p if the agent does not have the ability to operate on p. An agent a cannot have a duty against b to forbear on p if a or b is not able to operate on p.
- Axioms for consistency of claims tables
- S5 axioms for knowledge
- Comparison axioms for utilities
- PDL axioms for actions
- Rules of inference: Modus Ponens, Knowledge Generalization and Action Generalization.

Setting an Agenda

- How do legal claims come into being? In society, presumably through ownership relations that are established by acts of giving, selling, inheriting, etcetera. We have already seen in the examples that there is a simple way to incorporate some of this in our model. How can this be extended?
- It may be interesting to distinguish between actions of agents and acts of nature. Acts of nature could destroy value (described in terms of some payoff function), so that acts of repair are needed to restore the situation (get better payoff again).
- This makes for interesting situations, for now work may be needed, and all the legal issues concerning who is responsible for the work show up.

- Allowing acts of nature gives the right of a against b for a not to operate on p more bite, for this could mean now that a is exempt from the work involved in "repairing" p after an act of nature that has "destroyed" p.
- But this means we have to extend to legal claims that have a conditional/temporal flavour: duties to restore a situation whenever it is corrupted, privileges to react in certain ways whenever another agent gives you occasion to do so ...

Conclusion

Philosophical discussions about ethics too often suggest that reasoning about how people should behave either must be phrased fully in terms of duties and rights or fully in terms of personal and societal interests. In fact, actual moral or legal debate covers both of these aspects, and deals with the interaction between them.

It is our hope that now that we have brought the two together in a single formal model, admittedly too simple to be true to life but hopefully not too simple to be useful for abstract modelling, we can make further progress in the direction of more realistic modelling of moral and legal debate.

Connection with Natural Language Processing: the Flint initiative [HvdSvD$^+$] ("A language for formally describing the interpretation of laws concerning the admission of Aliens into The Netherlands").

More Links to the Literature

- Earlier systems of deontic logic inspired byHohfeld: [Kan72, Mak86].
- Papers on propositional control (action in the world as ability to change basic facts): [Eij00, vdHW05].
- Papers Connecting Deontic, Epistemic and Dynamic Logic: [Mey88, Bro04, SD15, vdM96, JCL15, PPC06, Wan05]
- Literature on Formal Contract Analysis: [PS07]
- Formal Modelling of Legal Argument: [Kow95, Pra13, HvdSvD$^+$]

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